Novel BPS Wilson Loops in Quiver Chern-Simons-matter theories

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- The VEV of a circular half-BPS Wilson loop can be computed exactly using localization at finite 't Hooft coupling and finite N *[Pestun, 07]*.

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- The VEV of a circular half-BPS Wilson loop can be computed exactly using localization at finite 't Hooft coupling and finite N *[Pestun, 07]*.
- This VEV is a nontrivial function of the coupling constant, interpolating between weak coupling results from perturbative field theory and strong coupling results (*in Large N limit*) from gravity side *[Berestein etal, 98][Drukker, Gross, Ooguri 99]*.

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- It is holographically dual to M-theory on $AdS_4 * S^7/Z_k$ or type IIA string theory on $AdS_4 \times CP^3$.

Wilson loops in ABJM theory - I

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- Such Wilson loop was $1/6$ BPS when it is along a line or a circle. It is closely related to $1/2-$ ($1/3$-)BPS Wilson loop in generic $\mathcal{N} = 2(3)$ Chern-Simons-matter (CSM) theories in *[Gaiotto, Yin (GY), 07]*.

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- To provide a matrix model calculation for the VEV of this $1/6$ BPS Wilson loop was the original motivation for *Kapustin, Willett and Yaakov* to develop the localization in 3d CSM theories.
  [Marino, lecture notes, 11]

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- Such loop was finally constructed by *Drukker and Trancanelli (DT) in 2009* by including the fermions in the construction and build a super-connection.
- This construct was elegantly explained by *K. Lee and S. Lee in 2010* via the *Brout-Englert-Higgs* mechanism.

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- $2/5$-BPS DT-type Wilson loops in $\mathcal{N}=5$ CSM theories (*[Hosomichi, Lee^3, Park, 0806], [ABJ, 08]*) were found by *K. Lee and S. Lee*.
- $1/2$-BPS DT-type Wilson loops in $\mathcal{N}=4$ CSM theories*[HLLLP, 0805][for $\mathcal{N}=4$ orbifold ABJM theory, Benna etal, 08]* were constructed in *[Ouyang, JW, Zhang, 1506][Cooke, Drukker, Trancanelli, 1506]*.

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- DT type Wilson loops also seem to preserve more supersymmetries than the GY type Wilson loops when they are along the same contour.
- Taken home message of this talk is that both these two speculations are *incorrect*.

$\mathcal{N}=2$ quiver CSM theories - vector multiplets

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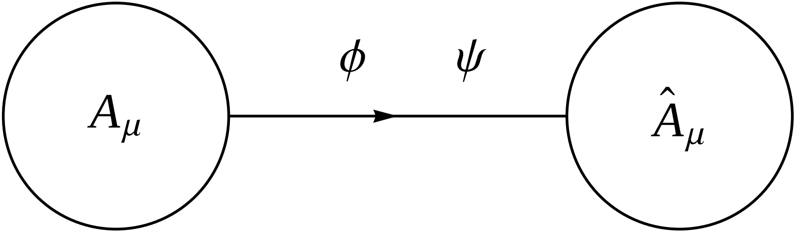
- Let us pick two adjacent nodes in the quiver diagram and the corresponding gauge group are $U(N_1)$ and $U(N_2)$. The Chern-Simons levels are k_1 and k_2, respectively.
- The vector multiplet for gauge group $U(N_1)$ include A_μ, σ, χ, D and the last three fields are the auxiliary fields. Similarly for gauge group $SU(N_2)$ we have the vector multiplet $\hat{A}_\mu, \hat{\chi}, \hat{\sigma}, \hat{D}$.

$\mathcal{N} = 2$ quiver CSM theories - chiral multiplets

- The chiral multiplet in the bifunamental representation of $U(N_1) \times U(N_2)$ includes the scalar ϕ, the spinor ψ and the auxiliary field F.



Supersymmetry transformation

- For the vector multiplet part, we only need the off-shell supersymmetry transformation of $A_\mu, \sigma, \hat{A}_\mu, \hat{\sigma}$ is,

$$\delta A_\mu = \frac{1}{2}(\bar{\chi}\gamma_\mu\theta + \bar{\theta}\gamma_\mu\chi), \quad \delta\sigma = -\frac{\mathrm{i}}{2}(\bar{\chi}\theta + \bar{\theta}\chi),$$
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- For the matter part we only need the off-shell supersymmetry transformation of ϕ and ψ

$$\begin{aligned}
\delta\phi &= \mathrm{i}\bar{\theta}\psi, \quad \delta\bar{\phi} = \mathrm{i}\bar{\psi}\theta, \\
\delta\psi &= (-\gamma^\mu D_\mu\phi - \sigma\phi + \phi\hat{\sigma})\theta + \mathrm{i}\bar{\theta}F, \\
\delta\bar{\psi} &= \bar{\theta}(\gamma^\mu D_\mu\bar{\phi} + \hat{\sigma}\bar{\phi} - \bar{\phi}\sigma) - \mathrm{i}\theta\bar{F},
\end{aligned} \quad (2)$$

GY type BPS Wilson loops

- In Minkowski spacetime, one can construct a GY type 1/2 BPS Wilson loop along an infinite straight line $x^\mu = \tau\delta^\mu_0$ as

$$W_{\mathrm{GY}} = \mathcal{P}\exp\left(-\mathrm{i}\int \mathrm{d}\tau L_{\mathrm{GY}}(\tau)\right),$$
$$L_{\mathrm{GY}} = \begin{pmatrix} A_\mu\dot{x}^\mu + \sigma|\dot{x}| & \\ & \hat{A}_\mu\dot{x}^\mu + \hat{\sigma}|\dot{x}| \end{pmatrix}. \qquad (3)$$

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- The preserved SUSY can be denoted as

$$\gamma_0\theta = {\rm i}\theta, \quad \bar{\theta}\gamma_0 = {\rm i}\bar{\theta}. \qquad (4)$$

DT-type BPS Wilson loops - I

- We can also construct the DT type Wilson loop

$$
\begin{aligned}
W_{\mathrm{DT}} &= \mathcal{P}\exp\left(-\mathrm{i}\int \mathrm{d}\tau L_{\mathrm{DT}}(\tau)\right), \\
L_{\mathrm{DT}} &= \begin{pmatrix} \mathcal{A} & \bar{f}_1 \\ f_2 & \hat{\mathcal{A}} \end{pmatrix}, \\
\mathcal{A} &= A_\mu \dot{x}^\mu + \sigma|\dot{x}| + m\phi\bar{\phi}|\dot{x}|, \quad \bar{f}_1 = \bar{\zeta}\psi|\dot{x}|, \\
\hat{\mathcal{A}} &= \hat{A}_\mu \dot{x}^\mu + \hat{\sigma}|\dot{x}| + n\bar{\phi}\phi|\dot{x}|, \quad f_2 = \bar{\psi}\eta|\dot{x}|.
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- To make it preserve the SUSY in (4) at least classically, it is enough to require that *[K. Lee, S. Lee, 2010]*

$$\delta L_{\mathrm{DT}} = \partial_\tau G + \mathrm{i}[L_{\mathrm{DT}}, G], \qquad (6)$$

for some Grassmann odd matrix

$$G = \begin{pmatrix} & \bar{g}_1 \\ g_2 & \end{pmatrix}. \qquad (7)$$

DT-type Wilson loops - II

- We find that the necessary and sufficient conditions for the existence of such $\bar{g}_1$ and g_2 are

$$\bar{\zeta}^{\alpha} = \bar{\alpha}(1, \mathrm{i}), \quad \eta_{\alpha} = (1, -\mathrm{i})\beta,$$
$$m = n = 2\mathrm{i}\bar{\alpha}\beta. \tag{8}$$

Such DT type Wilson loop is 1/2 BPS, and the preserved SUSY is the same as (4). Note that there are two free complex parameters $\bar{\alpha}$ and β in the Wilson loop, and they can be any complex constants. When $\bar{\alpha} = \beta = 0$, it becomes the GY type Wilson loop.

Generalizations

- There could be other matters couple to these two gauge fields. They will change the on-shell values of σ and $\hat{\sigma}$ in the Wilson loops we will construct. The structure of these Wilson loops will not be changed.

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- This construction can be also applied to the case when $U(N)$ is replaced by $SO(N)$ or $USp(N)$, and the case when there are matter fields in the adjoint representation. For the last case, one just simply let $\hat{A}_\mu \equiv A_\mu$ and $\hat{\sigma} \equiv \sigma$.

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- The case with multi matter fields in the bifundamental and anti-bifundamental representations were also considered. The DT type Wilson loops can be divided into four classes.

GY-type Wilson loops in ABJM theory - I

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- In ABJM theory, there are four scalars ϕ_I and four fermions ψ_I in the bifundamental representation.
- A general GY type Wilson loop along the timelike infinite straight line $x^\mu = \tau\delta_0^\mu$ takes the form

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\begin{aligned}
W_{\rm GY} &= \mathcal{P}\exp\left(-\mathrm{i}\int \mathrm{d}\tau L_{\rm GY}(\tau)\right),\\
L_{\rm GY} &= \begin{pmatrix} \mathcal{A}_{\rm GY} & \\ & \hat{\mathcal{A}}_{\rm GY} \end{pmatrix},\\
\mathcal{A}_{\rm GY} &= A_\mu \dot{x}^\mu + \frac{2\pi}{k} R^I{}_J \phi_I \bar{\phi}^J |\dot{x}|,\\
\hat{\mathcal{A}}_{\rm GY} &= \hat{A}_\mu \dot{x}^\mu + \frac{2\pi}{k} S_I{}^J \bar{\phi}^I \phi_J |\dot{x}|.
\end{aligned} \tag{9}
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GY-type Wilson loops in ABJM theory - I

- Up to some $SU(4)$ transformation, the only GY-type Wilson line preserving Ponincare supercharges are the ones with $R^I{}_J = S_J{}^I = \mathrm{diag}(-1,-1,1,1)$. They are $1/6$-BPS preserving the supersymmetries

$$\begin{aligned} &\gamma_0\theta^{12} = \mathrm{i}\theta^{12}, \quad \gamma_0\theta^{34} = -\mathrm{i}\theta^{34}, \\ &\theta^{13} = \theta^{14} = \theta^{23} = \theta^{24} = 0. \end{aligned} \tag{10}$$

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- This is just the Wilson loop that was constructed in *[DPY][Chen JW][RSY]*. Here we show that this is the only form of GY type 1/6 BPS Wilson loops up to some $SU(4)$ transformation. Especially, we find that we do not need to require that $R^I{}_J$ or $S_I{}^J$ is a hermitian matrix *a priori*, and we show that it is the result of supersymmetry.

Novel DT-type Wilson loops in ABJM theory - I

- We turn to constructing a DT type Wilson loop along a straight line that preserves at least the supersymmetries (10). A general DT type Wilson loop is

$$\begin{aligned} W_{\mathrm{DT}} &= \mathcal{P}\exp\left(-\mathrm{i}\int \mathrm{d}\tau L_{\mathrm{DT}}(\tau)\right), \\ L_{\mathrm{DT}} &= \begin{pmatrix} \mathcal{A} & \bar{f}_1 \\ f_2 & \hat{\mathcal{A}} \end{pmatrix}, \\ \mathcal{A} &= \mathcal{A}_{\mathrm{GY}} + \frac{2\pi}{k} M^I{}_J \phi_I \bar{\phi}^J |\dot{x}|, \\ \hat{\mathcal{A}} &= \hat{\mathcal{A}}_{\mathrm{GY}} + \frac{2\pi}{k} N_I{}^J \bar{\phi}^I \phi_J |\dot{x}|, \\ \bar{f}_1 &= \sqrt{\frac{2\pi}{k}}\bar{\zeta}_I \psi^I |\dot{x}|,\ f_2 = \sqrt{\frac{2\pi}{k}}\bar{\psi}_I \eta^I |\dot{x}|. \end{aligned} \tag{12}$$

Novel DT-type Wilson loops in ABJM theory - II

- The supersymmetry conditions give that

$$\begin{aligned}
\bar{\zeta}_{1,2} &= \bar{\alpha}_{1,2}\bar{\zeta}, \quad \bar{\zeta}^{\alpha} = (1, \mathrm{i}), \\
\bar{\zeta}_{3,4} &= \bar{\gamma}_{3,4}\bar{\mu}, \quad \bar{\mu}^{\alpha} = (1, -\mathrm{i}), \\
\eta^{1,2} &= \eta\beta^{1,2}, \quad \eta_{\alpha} = (1, -\mathrm{i}), \\
\eta^{3,4} &= \nu\delta^{3,4}, \quad \nu_{\alpha} = (1, \mathrm{i}),
\end{aligned} \tag{13}$$

$$M^{I}{}_{J} = N_{J}{}^{I} = 2\mathrm{i}\begin{pmatrix} \bar{\alpha}_2\beta^2 & -\bar{\alpha}_2\beta^1 & & \\ -\bar{\alpha}_1\beta^2 & \bar{\alpha}_1\beta^1 & & \\ & & \bar{\gamma}_4\delta^4 & -\bar{\gamma}_4\delta^3 \\ & & -\bar{\gamma}_3\delta^4 & \bar{\gamma}_3\delta^3 \end{pmatrix}. \tag{14}$$

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- Class II
$$\bar{\alpha}_{1,2} = \beta^{1,2} = 0. \tag{16}$$
- Class III
$$\beta^{1,2} = \delta^{3,4} = 0. \tag{17}$$
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- Generically these Wilson loops preserving the same SUSY as GY type Wilson loops, i. e., they are $1/6$-BPS.
- In class I and II, for special parameters in the constructions, the Wilson loops become the half-BPS ones found by Drukker and Trancanelli. Our novel DT type Wilson loops include $1/6$-BPS GY type and half-BPS DT type ones as special cases.

DT type Wilson loops in $\mathcal{N} = 3, 4$ Chern-Simons-matter theories

- We found similar pattern in $\mathcal{N} = 4$ CSM theories: generally the DT type BPS Wilson loop along a straight line/circle is $1/4$ BPS, the same as GY type Wilson loops. For special parameters, supersymmetries preserved by the loops are enhanced to half-BPS.

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- For $\mathcal{N}=3$ CSM theories, the DT type BPS Wilson loop along a straight line/circle is $1/3$-BPS. there is no supersymmetry enhancement here. This is consistent with the results from the dual M-theory side *[Chen, JW, Zhu, 14]*.

Conclusion

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- We constructed DT-type Wilson loops in general $\mathcal{N}=2$ quiver CSM theories.
- Along a straight line or a circle, the generic DT-type Wilson loops in *ABJM* theory are $1/6$-BPS, which include $1/6$-BPS GY-type Wilson loops and half-BPS DT-type Wilson loops as special case.

Conclusion

- We constructed DT-type Wilson loops in general $\mathcal{N}=2$ quiver CSM theories.
- Along a straight line or a circle, the generic DT-type Wilson loops in *ABJM* theory are $1/6$-BPS, which include $1/6$-BPS GY-type Wilson loops and half-BPS DT-type Wilson loops as special case.
- Generically, these $1/6$-BPS DT-type Wilson loops are not locally $SU(3)$ invariant. This is different from the Wilson loops constructed in *[Cardinali, Griguolo, Martelloni, Seminara, 12]*.

Discussions

- *Cooke, Drukker and Trancanelli* argued that classically half-BPS DT-type Wilson loops in $\mathcal{N} = 4$ CSM theories may not be truly BPS at the quantum level. Only special linear combination of these loops will be BPS at the quantum level.

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- Currently they are performing three-loop computations.

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- This leads to the following two questions:
- **Which linear combination is truly BPS at the quantum level?**
- **How to construct the holographical dual of the above Wilson loops?**

Thanks for Your Attention !